XXV. Marginal Probability

Discrete Case

- We have an experiment with two random variables, Y_1 and Y_2.
- The marginal probability functions of Y_1 and Y_2 are

$$\begin{aligned} p_1(y_1) &:= P(Y_1 = y_1) \\ &= \boxed{\sum_{y_2} p(y_1, y_2)} \\ p_2(y_2) &:= P(Y_2 = y_2) \\ &= \boxed{\sum_{y_1} p(y_1, y_2)}. \end{aligned}$$

(For each y_1, hold it constant and add up over all the y_2's. You get an answer that depends on y_1.)

Continuous Case

- The marginal density functions of Y_1 and Y_2 are

$$\begin{aligned} f_1(y_1) &:= \boxed{\int_{y_2=-\infty}^{y_2=\infty} f(y_1, y_2)\, dy_2} \\ f_2(y_2) &:= \boxed{\int_{y_1=-\infty}^{y_1=\infty} f(y_1, y_2)\, dy_1} \end{aligned}$$

Note that it doesn't make sense to say $P(Y_1 = y_1)$, because that would be 0. But the second line does translate to continuous distributions.

$f_2(y_2) = \int_{y_1=-\infty}^{y_1=\infty} f(y_1, y_2)\, dy_1 =$ area of a slice.

Warning: Note the subscript change: $f_1(y_1) = \int_{y_2} f\, dy_2$, etc.

Integrate over whatever your range for y_2 is.

Example I

Roll two dice, a red die and a blue die. Define the variables:

$$\begin{aligned} Y_1 &:= \text{what shows on red die} \\ Y_2 &:= \text{total} \end{aligned}$$

Compute the marginal probability function $p_1(y_1)$.

$$
\begin{aligned}
Y_1 &:= \text{what shows on red die} \\
Y_2 &:= \text{total} \\
p_1(y_1) &= ?? \\
p_1(1) &= p(1,2) + p(1,3) + \cdots + p(1,7) + p(1,8) + \cdots + p(1,12) \\
&= \frac{1}{36} + \cdots + \frac{1}{36} + 0 + \cdots + 0 \\
&= \boxed{\frac{1}{6}} \\
p_1(2) &= p(2,2) + p(2,3) + p(2,4) + \cdots + p(2,8) + p(2,9) + \cdots + p(2,12) \\
&= 0 + \frac{1}{36} + \cdots + \frac{1}{36} + 0 + \cdots + 0 \\
&= \boxed{\frac{1}{6}} \\
&\vdots \\
p_1(6) &= \boxed{\frac{1}{6}}
\end{aligned}
$$

Example II

As in Example I, roll two dice, a red die and a blue die. Define the variables:

$$
\begin{aligned}
Y_1 &:= \text{what shows on red die} \\
Y_2 &:= \text{total}
\end{aligned}
$$

Compute the marginal probability function $p_2(y_2)$.

$$
\begin{aligned}
p_2(2) &= p(1,2) + p(2,2) + \cdots + p(6,2) = \frac{1}{36} + 0 + \cdots + 0 = \boxed{\frac{1}{36}} \\
p_2(3) &= p(1,3) + p(2,3) + \cdots + p(6,3) = \boxed{\frac{2}{36}} \\
p_2(4) &= \boxed{\frac{3}{36}} \\
&\vdots \\
p_2(7) &= \boxed{\frac{6}{36}} \\
\text{[They'll biff this line:]} \quad p_2(8) &= \boxed{\frac{5}{36}} \\
&\vdots \\
p_2(12) &= \boxed{\frac{1}{36}}
\end{aligned}
$$

Example III

Consider the joint density function

$$f(y_1, y_2) := 6(1 - y_2), 0 \le y_1 \le y_2 \le 1.$$

Find the marginal density function $f_1(y_1)$.

(Graph.)

$$f_1(y_1) = \int_{y_2=y_1}^{y_2=1} 6(1 - y_2)\, dy_2 = \boxed{3y_1^2 - 6y_1 + 3}$$

We'll use this for Example II in the next lecture.

Example IV

As in Example III, consider the joint density function

$$f(y_1, y_2) := 6(1 - y_2), 0 \le y_1 \le y_2 \le 1.$$

Find the marginal density function $f_2(y_2)$.

(Graph.)

$$f_2(y_2) = \int_{y_1=0}^{y_1=y_2} 6(1 - y_2)\, dy_1 = \boxed{6y_2 - 6y_2^2}$$

We'll use these again in the next lecture.

Example V

Consider the joint density function

$$f(y_1, y_2) := \begin{cases} e^{-y_2}, & 0 \le y_1 \le y_2 < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the marginal density function $f_1(y_1)$.

(Graph.)

$$\begin{aligned} f_1(y_1) &= \int_{y_2=y_1}^{y_2=\infty} e^{-y_2}\, dy_2 \\ &= \left(-e^{-y_2}\right)\Big|_{y_2=y_1}^{y_2=\infty} \\ &= \boxed{e^{-y_1}} \end{aligned}$$

We'll use this for Example V in the next lecture.