X. Markov's Inequality

Markov's Inequality

- Markov's inequality is a quick way of estimating probabilities based on the mean of a random variable.
- Suppose Y is a random variable with only positive values, and a is constant.

$$\boxed{\textbf{Markov's Inequality}: P(Y \geq a) \leq \frac{E(Y)}{a}}$$

- We can reverse it:

$$\boxed{P(Y < a) \geq 1 - \frac{E(Y)}{a}}$$

Emphasize that this gives you one sided bounds, not exact equalities.

Example I

Surveys show that students on a particular campus carry an average of \$20 in cash. If you meet a student at random, estimate the chance that she is carrying more than \$100. Also estimate the chance that she is carrying less than \$80.

$$
\begin{aligned}
P(Y \geq 100) &\boxed{\leq} \frac{E(Y)}{100} = \frac{20}{100} = \boxed{\frac{1}{5}} \\
P(Y \geq 80) &\leq \frac{E(Y)}{80} = \frac{1}{4} \\
P(Y > 80) &\boxed{>} \boxed{\frac{3}{4}}
\end{aligned}
$$

Example II

A rental car agency determines that customers renting cars for a week put an average of 210 miles on the cars. A customer has just rented a car for a week. Estimate the probability that he will put more than 350 miles on the car.

Markov says

$$
\begin{aligned}
P(Y \geq a) &\leq \frac{E(Y)}{a} \\
P(Y \geq 350) &\boxed{\leq} \frac{210}{350} = \frac{3}{5} = \boxed{60\%}
\end{aligned}
$$

Example III

Seismic data indicate that California suffers a major earthquake on average once every 10 years. What can we say about the probability that there will be an earthquake in the next 30 years?

Let Y := the waiting time until the next earthquake.

Markov says that $P(Y \geq 30) \leq \dfrac{E(Y)}{30} = \dfrac{1}{3}$, so the probability that there will be one is $\boxed{\geq \dfrac{2}{3}}$.

Example IV

A factory that produces batches of 1,000 laptops each finds that on average, two laptops per batch are defective. Estimate the probability that fewer than five laptops in the next batch will be defective.

Markov says

$$
\begin{aligned}
P(Y \geq a) &\leq \frac{E(Y)}{a} \\
P(Y \geq 5) &\leq \frac{2}{5} = 40\% \\
P(Y < 5) &\geq 60\%
\end{aligned}
$$

So $\boxed{\text{there is at least a 60\% chance that fewer than 5 laptops will be defective}}$.

Example V

A grocery store sells an average of 30 cans of tuna per day. Estimate the probability that it will sell more than 80 cans tomorrow.

Markov says

$$\begin{aligned} P(Y \geq a) &\leq \frac{E(Y)}{a} \\ P(Y \geq 80) &\boxed{\leq} \frac{30}{80} = \frac{3}{8} = \boxed{37.5\%} \end{aligned}$$